

Hidden in plain sight. The Philosopher's Star.

An alchemical excursus on magic stars of the fifth order.

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Introduction

Magic squares are mathematical constructs in which the sum of the numbers in each row, column, and diagonal is the same. The order of a magic square is determined by the number of cells in a row and any set of numbers can be used, as long as they meet the condition of constant row/column summation. In particular, natural magic squares utilize only natural numbers from 1 to n in their formation.

In the past, magic squares have been utilized in various forms of charms, talismans, and other objects associated with magic. They have also been observed in philosophical or alchemical speculations, serving as expressions of certain ideas or instructions for experiments. Today, magic squares are studied as mathematical recreations and as problems in number theory, but they have held deeper significance beyond mathematical questions. This may be attributed to their mysterious nature, which suggests the operation of some supernatural intelligence. Regardless of the reason, magic squares are a prime example of the intrinsic harmony of numbers and serve as an interpreter for mankind of the cosmic order that pervades all existence [1,2].

In the realm of recreational mathematics, magic stars have received significantly less attention than magic squares and cubes. A pure magic star is a set of integers from 1 to $2n$ placed at the $2n$ exterior points of intersection of the lines that form a regular polygram, such that the sum of the four integers found in any of the n lines is a constant value, known as the magic sum. The magic sum is given by: $S = 4n+2$, where n is the order of the star. The smallest examples of normal magic stars are 6-pointed, with some examples being magic hexagrams ($n = 6$), magic heptagrams ($n = 7$), and so on [3,4].

It is important to note that no star polygons with fewer than 5 points exist, and the construction of a normal 5-pointed magic star is impossible. This five-pointed star, or pentagram, cannot be constructed using the integers from 1 to 10, being the lowest possible configuration achieved by using the numbers from 1 to 12, with the exclusion of 7 and 11. The magic sum, in this case, is 24. Another set of solutions is obtainable from the same series of 1 to 12, but leaving out the 2 and 6, with a magic sum of 28.

A method for constructing magic pentagonal stars has been described in the encyclopedic work of WS Andrews. This method, which was first presented in an article in *The Monist*, allows for the construction of pentagonal stars using a set of five numbers that need not be in an arithmetic series [5,6]

Given that it is not possible to construct a pentagonal star using consecutive numbers from 1 to 10, an interesting question emerges:

- Is it possible to construct a magic pentagonal star (a pentagram) using the minimal series of numbers from 1 to 9, with the minimal number of repetitions? (Ideally, only one number repeated).
- If this is possible, is there any relationship between the resulting pentagonal star and the 3x3 magic square?

As we will see, the answer to both questions is affirmative. In the following sections, a brief review about the history of magic squares and the pentagram, as well as their relationship with western alchemical tradition, will precede our demonstration.

The journey of magic squares, from East to West

The Lo Shu

The Lo Shu square is undoubtedly the oldest known magic square. The earliest indisputable reference to its numbers in regular order dates back to the first century AD, however, indirect references can be traced back to the writings of Tsou Yen, an alchemist and philosopher active in the fourth century. For the ancient Chinese, the Lo Shu represented a microcosm of the universe, an *imago mundi*, that was linked to both the heavens and the earth through the harmonious balance of its numbers, centered around the strong central number five. This central number was located at the axis mundi, the cosmic axis connecting these two worlds. After having been kept private or used in cults for over thirteen hundred years, the Lo Shu became widely known in China during the tenth century AD. By then, the mysteries of the Lo Shu had been fully absorbed into Chinese culture and were commonly used as a protective charm and a magical means of divination [2,7].

Evaluating the specifics of the communication flow through Central Asia is beyond the scope of this work, but it is clear that elements from various cultural traditions were combined and recombined in this region. Scholars, religious figures, and technicians traveled long distances, as evidenced by the case of early Islamic scholars. In contrast, scholars from the urban centers of the ancient Greek tradition played no key role in reviving the sciences from the 8th to the 10th century.

It is likely that not only knowledge on magic squares, but also the strange design pattern of square horoscopes that suddenly emerged in Western astrology during this period

came from East Asia. Due to cultural exchange on the Silk Road, the same three-by-three square diagram with diagonal divided corner fields, applied to the disposition of heavenly forces in a cyclical system of twelve regions, was found in both China and the Middle East. This further highlights the significance of the long-distance interaction between East and West [8].

Magic squares in India

The use of magic squares in India is exemplified by the generic Hindu temple form, which is based on a magic square of three, as the Indian tradition holds that a profound understanding of geometry and astronomy is necessary for the proper alignment and construction of buildings and temples, through the use of gnomons and magic squares in the form of the Vaastu Purusha mandala or yantra. This mandala, in the shape of a human figure lying face down, serves as a blueprint for the layout of the temple and its various spaces, with each square corresponding to a specific aspect of the human anatomy, as well as the universe at large. This alignment serves to reinforce the connection between the temple and the cosmos, being this yantra a representation of the idealized sanctuary of the mind and the connection between the microcosm of the individual and the macrocosm of the universe [1].

Magic squares in Islam

Magic squares were first introduced to the Western world through Islamic texts of the 10th century. The first published reference to a magic square in the Islamic world is believed to have been in an Arabic treatise attributed to Jasbir ibn Hayyan (known as Geber in Europe) around 900 A.D. This charm, known as the baduh seal, was presented as a numerical acrostic using the first nine numbers of the abjad system, which equates letters with corresponding numbers. This practice of calculating with letters was considered a secret science known only to "the authorities in divine learning." The Brothers of Purity, a Muslim brotherhood, further popularized the use of magic squares in the Islamic world through their encyclopedia, the *Rasa'il*, published in 989 A.D. These squares were described as small models of a harmonious universe. The first seven squares, with sides ranging from three to nine squares, were associated with the seven planets. [1]

Magic squares in Europe

Magic squares arrived in Europe relatively late, and the first evidence of magic squares in Europe is believed to have been in the work of Abraham ibn Ezra of Toledo in 12th-century Spain. The earliest set of European magic squares were likely derived from the Islamic planetary squares translated by Jewish and Christian scholars living in Spain during the 13th century. These squares are considered to be the source of the Islamic astrological and cosmological lore that contributed to the development of the Cabala [1].

Manuscripts as the *Picatrix* from the thirteenth century are particularly noteworthy. This seminal work on astronomy and astrology, which was translated from an Arabic original

written in the tenth century, introduced the first examples of magic squares up to the ninth order, but provided no instructions for their construction.

A deeper interest in magic squares can be observed from the fourteenth century onwards. At this time, Byzantine scholar Manuel Moschopoulos described the principles of constructing magic squares in a letter to a friend. He employed the Persian continuous and knight's move method for odd-order squares, and the Indian method for fourth and eight order squares. Despite these instructions, interest in magic squares did not significantly increase until the publication of Agrippa von Nettesheim's *Occulta Philosophia* in 1531 and Girolamo Cardano's mathematical treatise *Practica Arithmetice et Mensurandi Singularis* eight years later. In 1567, the *Archidoxa Magica*, attributed to Paracelsus, was published, with the seventh book describing talismans of the planets, made of corresponding metal and bearing engraved magic squares, with the same planetary and square sequence as in Agrippa's book. Both treatises received significant attention, with Agrippa's work being among the most printed books of the sixteenth century, but Cardano's book did not achieve the same level of popularity, nor did German mathematical works written by Michael Stifter in 1544 and Adam Riese around 1550.

The Pentagram

It is believed that the 9-Magic Square may have originated from the same ultimate source as the Pentagram badge of the Greek Pythagorean Brotherhood, which flourished in Crotona in Southern Italy from around 525 B.C. The Pentagram prominently appears on both sides of a spouted ewer found at Jemdet Nasr in southern Mesopotamia, which is currently housed in the Ashmolean Museum in Oxford. This ewer, dating from earliest Sumerian times (around 3000 B.C.), confirms the idea that Pythagorean philosophy was based on theories that were already present, and possibly even for millennia, in the Middle East. The pentagram is also associated with Arabic talismanic tradition, as seen in manuscripts such as “*al-Tabşirah fî ‘ilm al-nujûm*” by ‘Umar ibn Yūsuf, Sultan of Yemen, d. 1296 (Fig.1), and in Western Hermetic texts such as the well-known Agrippa's “*De Occulta Philosophia*” (Fig. 2, left) and Battista's “*Calendarium Naturale Magicum Perpetuum*” (Fig. 2, right), where it is associated with the concept of macrocosmos-microcosmos [9,10].



Fig.1. Pentagrams in “al-Tabṣīrah fī ‘ilm al-nujūm” by ‘Umar ibn Yūsuf, Sultan of Yemen, d. 1296. Bodleian Library MS. Huntington 233

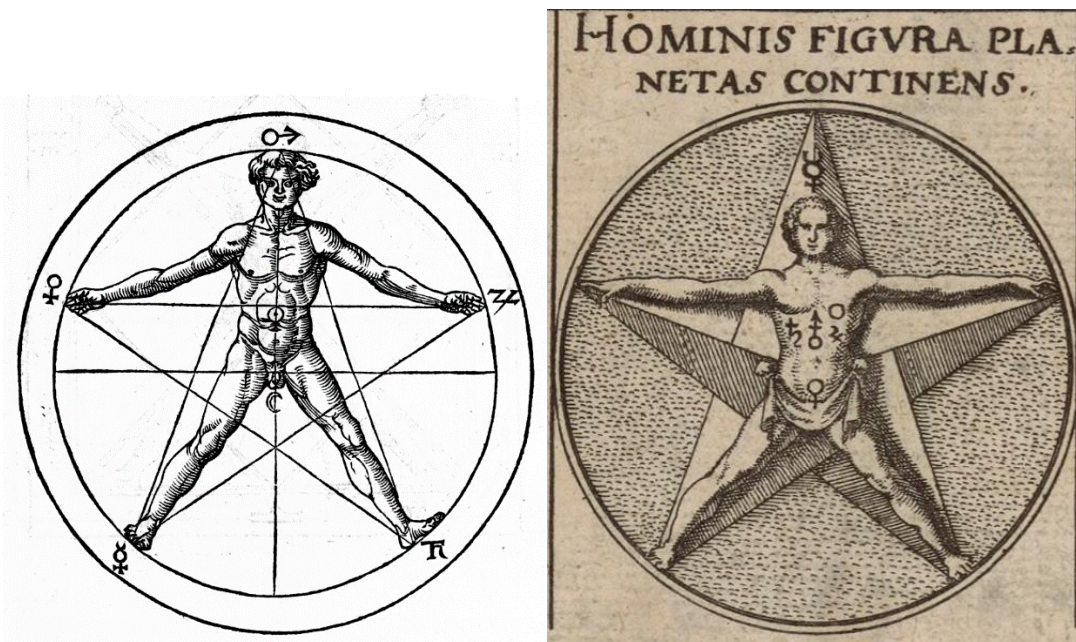


Fig. 2. Left, man inscribed in a pentagram, from Heinrich Cornelius Agrippa's *De Occulta Philosophia libri III*; right, man inscribed in a pentagram, from Johan Battista's *Calendarium Naturale Magicum Perpetuum*.

A hidden answer in alchemical works?

It is intriguing to consider that despite more than 2000 years of history of magic squares, and even more so in the last century, some questions have yet to be posed. But as Edgar Allan Poe said in "The Purloined Letter," *"The best place to hide is in plain sight"*.

As we have seen, magic squares and pentagrams are inherently connected to Western alchemical tradition. The knowledge of alchemy was kept secretive as practitioners believed it could only be fully understood by true adepts. This knowledge was often encoded in alchemical emblems.

An alchemical emblem is a collection of alchemical symbols arranged into a cohesive image, each symbol having a specific meaning within the context of the emblem. While these emblems may appear complex and even surreal at times, they aim to convey a narrative through the use of symbols and the way they are interconnected in the emblem's geometry.

We will examine in the following paragraphs some of the most well-known alchemical emblems in search of answers.

The "Senior Adolphus" image (Fig. 3) appears in the work "Azoth of the Philosophers" by Basil Valentine in 1613. The image depicts two philosophers, Senior and Adolphus, conversing at the base of a tree with three roots. Between them is a downward-pointing triangle, with Sulphur on the left vertex beside Senior, Salt on the right beside Adolphus, and Mercury at the central vertex. The tree has seven branches representing the planets, with the Sun on the left, followed by Mars and Venus, then Mercury at the center, followed by Saturn, Jupiter, and the Moon on the right. An upward-pointing triangle, which mirrors the one below, is also depicted in the branches of the tree.



Fig. 3. Basil Valentine's "Senior Adolphus" emblem.

The “Rebis” image (Fig. 4) in the same work (emblem 5) is also known for its representation of alchemical ideas. The image shows a double-headed hermaphrodite, with a male on the right side and a female on the left, standing within an egg-shaped space. The hermaphrodite is depicted standing on a winged and fire-breathing dragon, which in turn is perched upon a winged globe. Inside this winged globe, there is the union of the square of four and the triangle of three. The hermaphrodite is holding a compass in its right male hand and a square in its left female hand. The seven planets are also depicted around the hermaphrodite, with Mercury above its head, the Sun, Mars, and Venus on the male side, and the Moon, Jupiter, and Saturn on the female side.



Fig. 4. Basil Valentine's “Rebis” emblem.

"Metamorphosis Planetarum" by Johannes de Monte-Snyder is another notable alchemical work that contains one of the most extensive alchemical allegories and is immensely popular, many have struggled to understand its complexities, including the renowned scientist Isaac Newton. Its frontispiece (Fig. 5) will become relevant in the following sections.



Fig. 5. Monte Snyder's "Metamorphosis Planetarum" emblem.

Another two relevant emblems (Fig. 6) in our context appear in the “Book of Hieroglyphic Figures”, attributed to Nicolas Flamel and printed in 1612, although it is said to have been written in 1399, and in “Arithmologia sive de abditis Numerorum mysteriis”, written by Athanasius Kirtcher in 1665.

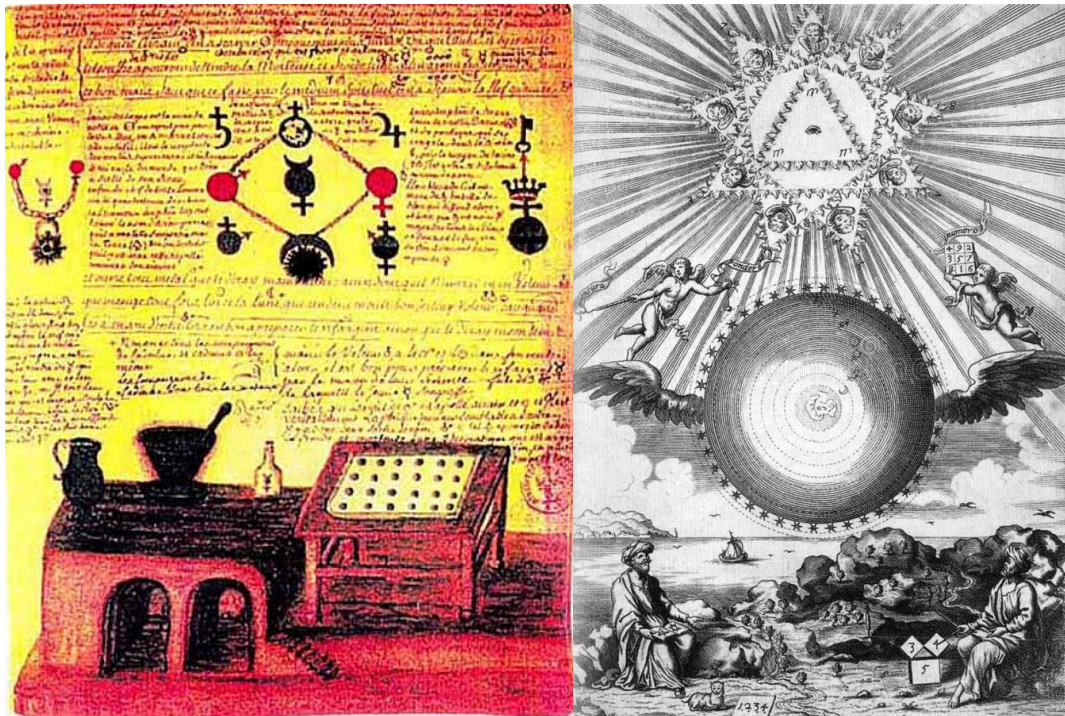


Fig. 6. Flamel's and Kitcher's emblems.

The secret key in Senior Adolphus

In the emblem of Senior Adolphus (Fig. 3), we observe two triangles: one upper, rectangular, containing the five ordinary metals and two noble metals, and one lower equilateral, with the tria prima at its vertices. It should be noted that Mercury appears in the upper triangle as an ordinary metal, while in the lower triangle it represents the Spiritual Mercury.

If we overlay the two triangles, as in the Masonic way that overlays the square and compass (curiously both appear in the Rebis), we obtain a pentagrammatic arrangement of the ten symbols (seven metals, and tria prima). Interestingly, in the inner pentagon we find the five planets (*hominis figura planetas continens*) while the outer pentagon contains the tria prima and the two noble metals, Gold and Silver, Sun and Moon, at the base (Fig. 7).

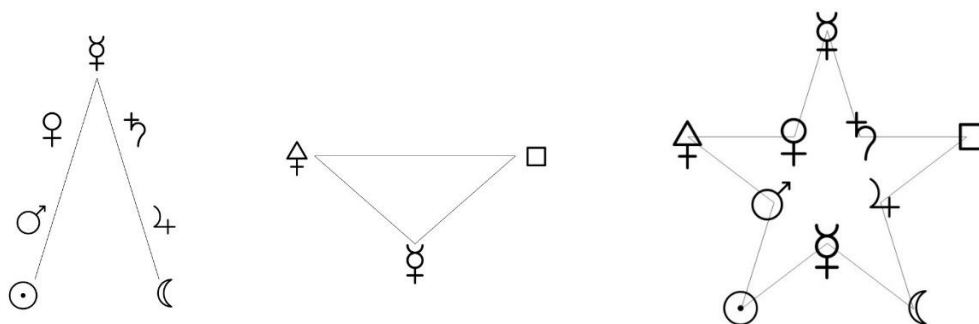


Fig. 7. Construction of the pentagrammatic arrangement based on "Senior Adolphus" emblem.

If we want to look for a connection with the square of Saturn, we have 10 elements to distribute in 9 cells. One possibility that we will explore later is to consider Mercury, ordinary metal, and Spiritual Mercury, as a single element in the square.

Another possibility, depicted in Fig. 8, following the well-known process as *alchemical marriage*, is to leave the two noble metals in a single cell. Going further with this option, it is not far-fetched to occupy the 5 cells corresponding to the magic square gnomon with the five ordinary metals, the three remaining cells on the edge with the tria prima, and Gold and Silver in the center. Thus, we have Mercury in diametrically opposite cells (*as above, so below*).

At this point, it is worth noting the hypothetical, but more than reasonable, connection with the concept of macrocosmos-microcosmos, both of the pentagram and the square of Saturn.

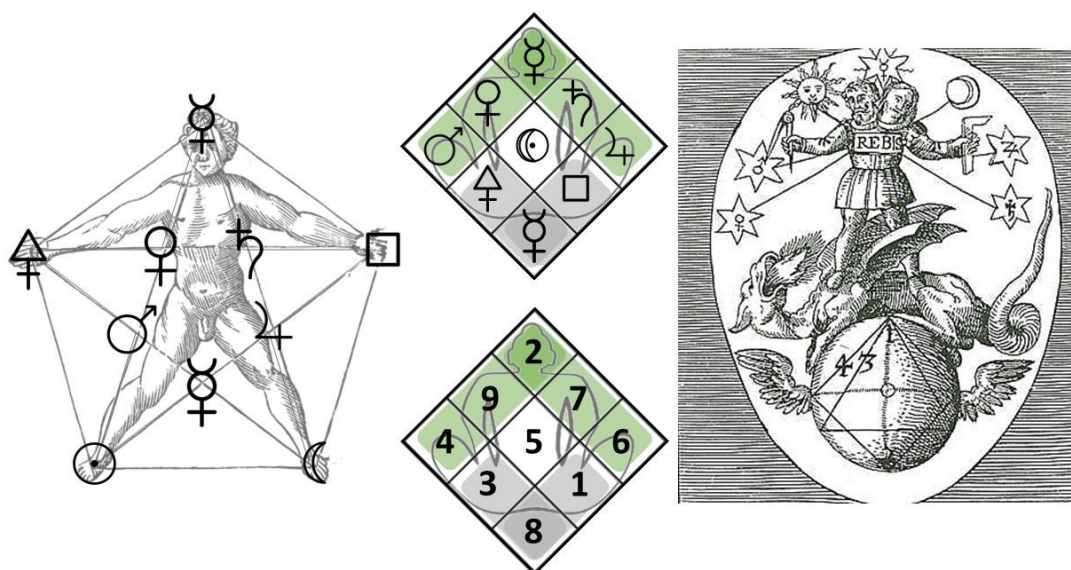


Fig 8. Relationship between the Pentagram and the square of Saturn.

The Philosopher's Star unveiled

If we associate each number of the magic square in its traditional arrangement, to the symbol occupying the same cell, we reach the pentagonal magic star of Fig. 9. **A pentagonal star with the numbers of the series from 1 to 9, repeating only the number 5, central element of the series, (and of the square), at the vertices of the base.** The lines of this pentagonal star add up to 20, the magic constant. For obvious reasons, we will call this star, and its variants by symmetry (inversion, rotation and reflection), **The Philosopher's Star.**

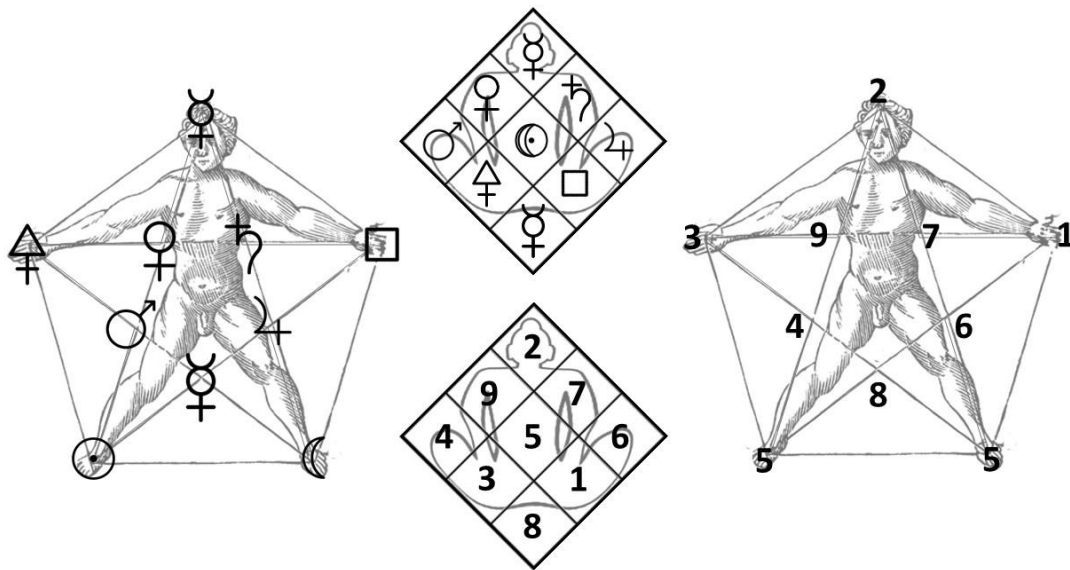


Fig. 9. The Philosopher's Star

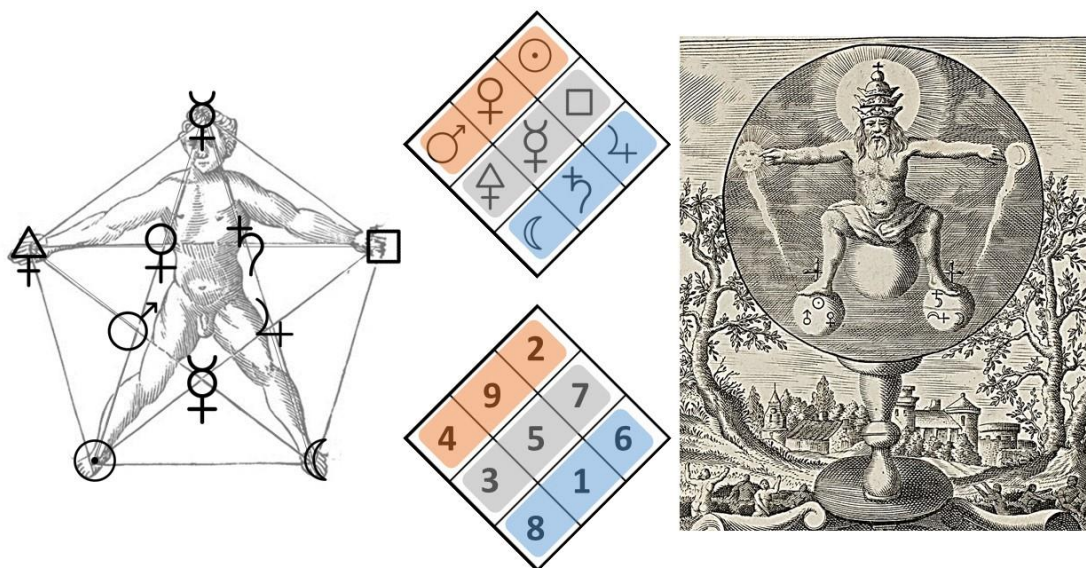


Fig. 10. A variant of The Philosopher's Star

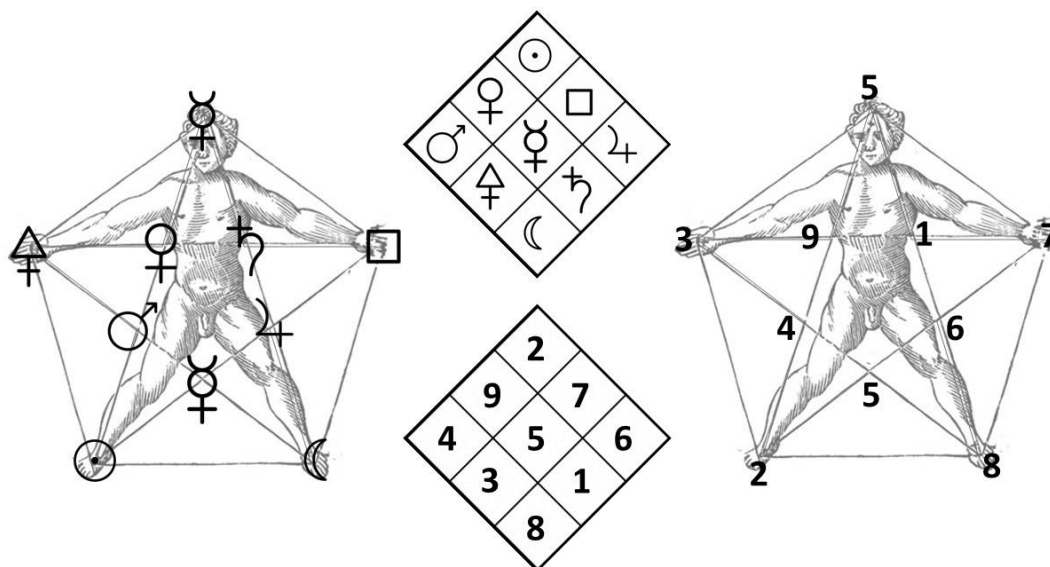


Fig. 11. Arithmologic diagram of the variant of The Philosopher's Star.

The other possible disposition to explore mentioned before is depicted in Figs. 10 and 11. In this case, Mercury is in the center, in its double quality of ordinary metal and Spiritual Mercury, flanked by Sulfur and Salt. The rest of the metals are thus separated according to alchemical tradition, as reflected in the emblems of "Rebis" and "Metamorphosis Planetarum".

Remarkably again, this disposition corresponds to one of the possible symmetry operations on the original Philosopher's Star, without altering the constant sum of the lines. We leave the reader to reflect on the possible arithmological and alchemical connections of what has been presented so far.

Infinite Philosopher's Stars

To know if this discovery is generalizable, we turn to the parameterization of the 3x3 magic square. In its most general form, without taking into account the diagonals, a magic square can be expressed as a sum table, reorganized.

In Fig. 12 some parametrizations of the 3x3 magic square are summarized, along with the constraint that enables the construction of the corresponding pentagonal star. As the reader can verify, the "Lucas parameterization", shown in the figure, always allows the construction of a pentagonal star.

In both, the general parametrization and the Lucas parametrization, the sum of the square is always 3 times the central cell, and that of the pentagonal star, 4 times the central cell. Do the 3 and 4 of the bottom of the "Rebis" emblem make a veiled reference to this fact? We may never know.

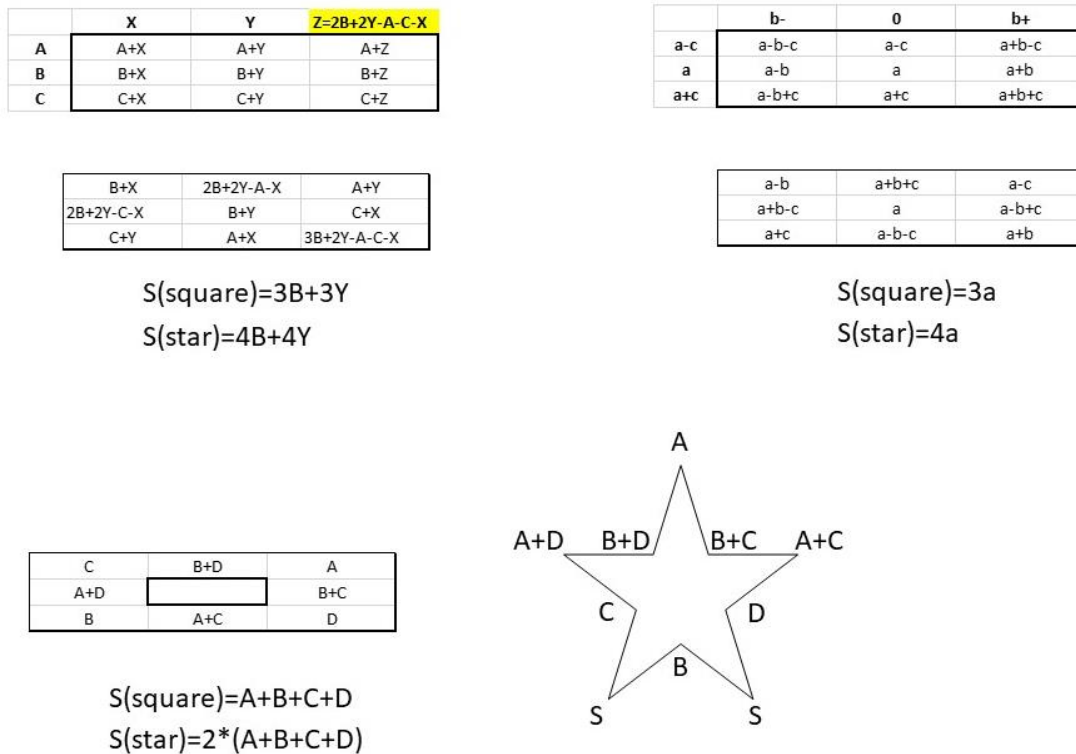


Fig. 12. Parametrizations of the 3x3 magic square that enable the construction of a Philosopher's Star.

A special case is the 3x3 hollow magic squares, whose parameterization is shown in the bottom of Fig. 12. Also in this case it is possible to build an associated pentagonal star, in which the only repeated number is the sum of the magic square.

There is also a relationship between the magic squares of order 4 and the stars of order 6 (Fig. 13), which will be the subject of study in future works.

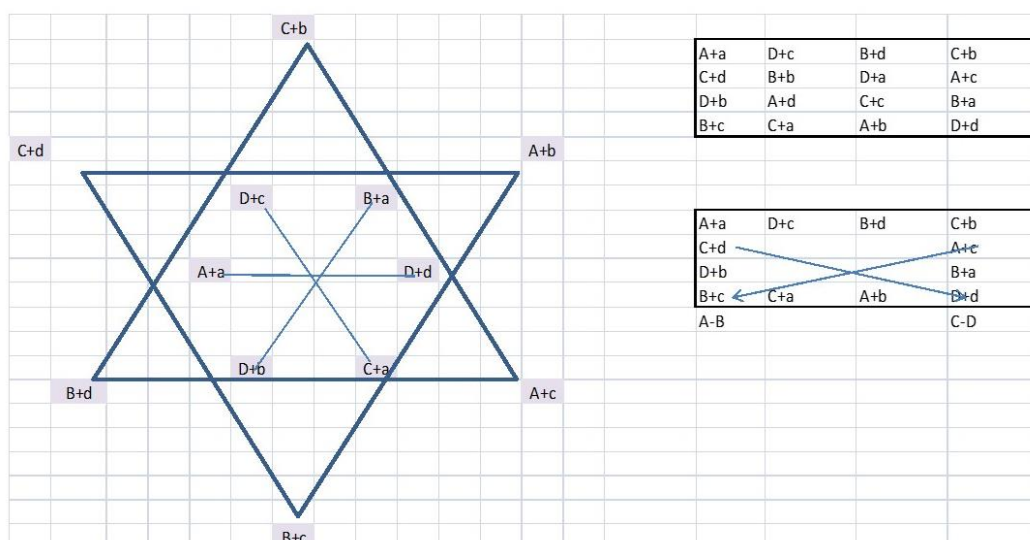


Fig. 13. A parametrization of the 4x4 magic square that enables the construction of a Philosopher's Hexagram.

Conclusions

Much of alchemical knowledge will remain hidden forever. However, the written legacy that has reached us today continues to feed the imagination of historians, scientists and scholars around the world.

It is this legacy that has allowed for the discovery of a **magic star of order 5 composed of the numbers 1 to 9, repeating the number 5 only once**. Despite the fact that there are no perfect order 5 stars, in the eyes of this author this solution is more perfect than existing solutions to date.

Furthermore, **a new construction method of pentagonal magic stars has been devised, based on magic squares of order 3**.

In honor of the work of the alchemists who inspired this discovery, we have named these stars **Philosopher's Stars**.

In future works, the connection between magic squares and stars of higher orders will be explored in depth.

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